

**Model Question Paper-02**

**Physical Sciences**  
**Test Booklet**

Sr. No \_\_\_\_\_  
Subject Code: 3

Time: 3:00 Hours

Maximum Marks: 200

**INSTRUCTIONS**

1. **You have opted for English as medium of Question Paper.** This Test Booklet contains one hundred and forty five (20 Part ‘A’ + 25 Part ‘B’ + 30 Part ‘C’) Multiple Choice Questions (MCQs). You are required to answer a maximum of 15, 20 and 20 questions form Part ‘A’ ‘B’ and ‘C’ respectively. If more than required number of questions are answered, only first 15, 20 and 20 questions in Part ‘A’ ‘B’ and ‘C’ respectively, will be taken up for evaluation.
2. Answer sheet has been provided separately. Before you start filling up your particulars, please ensure that the booklet contains requisite number of pages and that these are not torn or mutilated. If it is so, you may request the Invigilator to change the booklet. Likewise, check the answer sheet also. Sheets to rough work have been appended to the test booklet.
3. Write your Roll No., name, Your address and Serial Number of this Test booklet on the Answer sheet in the space provided on the side 1 of Answer sheet. Also put your signatures in the space identified.
4. **You must darken the appropriate circles with a pencil related to Roll Number, Subject Code, Booklet Code and Center Code on the OMR answer sheet. It is the sole responsibility of the candidate to meticulously follow the instructions given on Answer Sheet, failing which, the computer shall not be able to decipher the correct detail which may ultimately result in loss, including rejection of the OMR answer sheet.**
5. Each question in part ‘A’ carries 2 marks, Part ‘B’ 3.5 marks and Part ‘C’ 5 marks respectively. There will be negative marking @ 25% for each wrong answer.
6. Below each question in Part ‘A’, ‘B’ and ‘C’ four alternatives or responses are given. Only one of these alternatives is the “correct” option to the question. You have to find, for each question, the correct or the best answer.
7. Candidates found copying or resorting to any unfair means are liable to be disqualified from this and future examinations.
8. Candidates should not write anything anywhere except on answer sheet or sheets for rough work.
9. After the test is over, you MUST hand over the Test Booklet and the answer sheet (OMR) to the invigilator.
10. Use of calculator is not permitted.

Roll No.....

I have verified all the information  
filled in by the candidate

Name:.....

.....  
Signature of the Invigilator

**PART – A**

1. A granite block of  $2\text{ m} \times 5\text{ m} \times 3\text{ m}$  size is cut into 5 cm thick slabs of  $2\text{ m} \times 5\text{ m}$  size. These slabs are laid over a 2 m wide payment. What is the length of the pavement that can be covered with these slabs?

- 1. 100 m
- 2. 200 m
- 3. 300 m
- 4. 500 m

2. Which is the least among the following?

$0.33^{0.33}, 0.44^{0.44}, \pi^{-1/\pi}, e^{-1/e}$

- 1.  $0.33^{0.33}$
- 2.  $0.44^{0.44}$
- 3.  $\pi^{-1/\pi}$
- 4.  $e^{-1/e}$

3. What is the next number in this “see and tell” sequence?

1    11    21    1211    111221    \_\_\_\_\_

- 1. 312211
- 2. 1112221
- 3. 1112222
- 4. 1112131

4. A vertical pole of length  $a$  stands at the centre of a horizontal regular hexagonal ground of side  $a$ . A rope that is fixed taut in between a vertex on the ground and the tip of the pole has a length

- 1.  $a$
- 2.  $\sqrt{2a}$
- 3.  $\sqrt{3a}$
- 4.  $\sqrt{6a}$

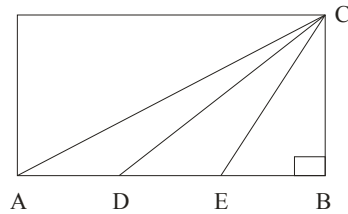
5. A peacock perched on the top of 12 m high tree spots a snake moving towards its hole at the base of the tree from a distance equal to thrice the height of the tree. The peacock flies towards the snake in a straight line and they both move at the same speed. At what distance from the base of the tree will the peacock catch the snake?

- 1. 16 m
- 2. 18 m
- 3. 14 m
- 4. 12 m

6. The cities of a country are connected by intercity roads. If a city is directly connected to an odd number of other cities, it is called an odd city. If a city is directly connected to an even number of other cities, it is called an even city. Then which of the following is impossible?

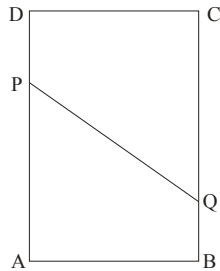
1. There are an even number of odd cities
2. There are an odd number of odd cities
3. There are an even number of even cities
4. There are an odd number of even cities

7. In the figure  $\angle ABC = \pi/2$ ,  $AD = DE = EB$ . What is the ratio of the area of triangle ADC to that of triangle CDB?



1. 1 : 1
2. 1 : 2
3. 1 : 3
4. 1 : 4

8. A rectangular sheet ABCD is folded in such a way that vertex A meets vertex C, thereby forming a line PQ. Assuming  $AB = 3$  and  $BC = 4$ , find PQ. Note that  $AP = PC$  and  $AQ = QC$



1.  $13/4$
2.  $15/4$
3.  $17/4$
4.  $19/4$

9. A string of diameter 1 mm is kept on a table in the shape of a close flat spiral i.e. a spiral with no gap between the turns. The area of the table occupied by the spiral is  $1\text{m}^2$ . Then the length of the string is

1. 10 m
2.  $10^2$  m
3.  $10^3$  m
4.  $10^6$  m

10. 25% of 25% of a quantity is  $x\%$  of the quantity where  $x$  is

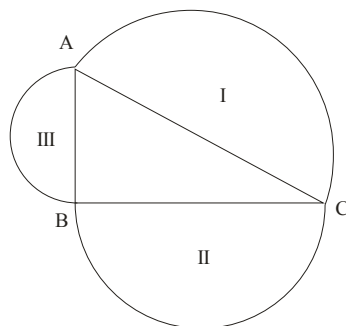
- |          |          |
|----------|----------|
| 1. 6.25% | 2. 12.5% |
| 3. 25%   | 4. 50%   |

11. In sequence  $\{a_n\}$  every term is equal to the sum of all its previous terms. If  $a_0 = 3n$  then

$$\lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} \text{ is}$$

- |      |      |
|------|------|
| 1. 3 | 2. 2 |
| 3. 1 | 4. e |

12. In the figure below. Angle  $ABC = \pi/2$ . I, II, III are the areas of semicircles on the sides opposite angles B, A and C, respectively. Which of the following is always true?

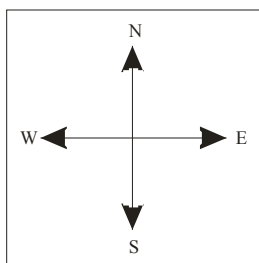


- |                         |                   |
|-------------------------|-------------------|
| 1. $II^2 + III^2 = I^2$ | 2. $II + III = I$ |
| 3. $II^2 + III^2 > I^2$ | 4. $II + III < I$ |

13. What is the minimum number of days between one Friday the 13<sup>th</sup> and the next Friday the 13<sup>th</sup>? (Assume that the year is a leap year).

- |       |       |
|-------|-------|
| 1. 28 | 2. 56 |
| 3. 91 | 4. 84 |

14. Suppose a person A is at the North-East corner of a square (see the figure below). From that point he moves along the diagonal and after covering  $1/3^{\text{rd}}$  portion of the diagonal, he goes to his left and after sometime he stops, rotates  $90^\circ$  clockwise and moves straight. After a few minutes he stops, rotates  $180^\circ$  anticlockwise. Towards which direction he is facing now?



- 1. North-East
- 2. North-West
- 3. South-East
- 4. South-West

15. Cucumber contains 99% water. Ramesh buys 100 kg of cucumbers. After 30 days of storing, the cucumbers lose some water. They now contain 98% water. What is the total weight of cucumbers now?

- 1. 99 kg
- 2. 50 kg
- 3. 75 kg
- 4. 2 kg

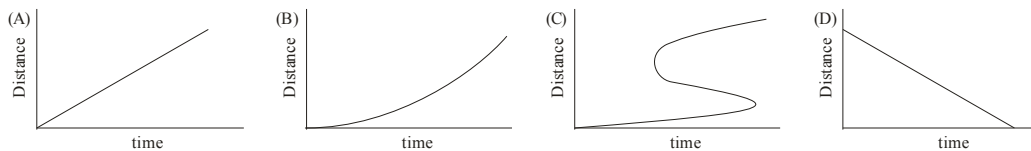
16. In a museum there were odd coins with their respective years engraved on them, as follows.

- A. 1837 AD
- B. 1907 AD
- C. 1947 AD
- D. 200 BC

Identify the fake coins(s)

- 1. Coin A
- 2. Coin D
- 3. Coins A and B
- 4. Coin C

17. A student observes the movement of four snails and plots the graphs of distance moved as a function of time as given in figures (A), (B), (C) and (D).



Which of the following is not correct?

- 1. Graph (A)
- 2. Graph (B)
- 3. Graph (C)
- 4. Graph (D)

18. Find the missing letter:

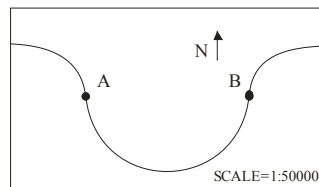
A	EGK	C
?		P
U		R
Q		V
B	OJF	D

- 1. H
- 2. L
- 3. Z
- 4. Y

19. Consider the following equation  $x^2 + 4y^2 + 9z^2 = 14x + 28y + 42z + 147$ , where  $x, y$  and  $z$  are real numbers. Then the value of  $x + 2y + 3z$  is

- 1. 7
- 2. 14
- 3. 21
- 4. Not unique

20. The map given below shows a meandering river following a semi-circular path, along which two villages are located at A and B. The distance between A and B along the east-west direction in the map is 7 cm. What is the length of the river between A and B in the ground?



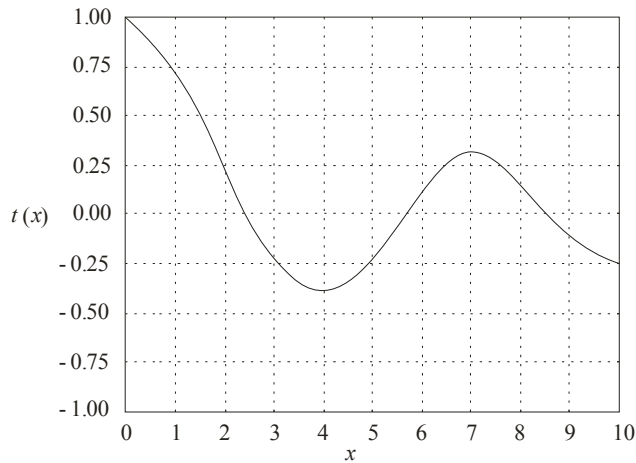
- 1. 1.1 km
- 2. 3.5 km
- 3. 5.5 km
- 4. 11.0 km

## PART – B

21. A  $2 \times 2$  matrix A has eigenvalues  $e^{i\pi/5}$  and  $e^{i\pi/6}$ . The smallest value of  $n$  such that  $A^n = I$  is

1. 20  
2. 30  
3. 60  
4. 120

22. The graph of the function  $f(x)$  shown below is best described by



1. The Bessel function  $J_0(x)$   
2.  $\cos x$   
3.  $e^{-x} \cos x$   
4.  $\frac{1}{x} \cos x$

23. In a series of five Cricket matches, one of the captains calls “Heads” every time when the toss is taken. The probability that he will win 3 times and lose 2 times is

1.  $1/8$   
2.  $5/8$   
3.  $3/16$   
4.  $5/16$

24. The unit normal vector at the point  $\left(\frac{a}{\sqrt{3}}, \frac{b}{\sqrt{3}}, \frac{c}{\sqrt{3}}\right)$  on the surface of the ellipsoid

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1, \text{ is}$$

1.  $\frac{bc\hat{i} + ca\hat{j} + ab\hat{k}}{\sqrt{a^2 + b^2 + c^2}}$   
2.  $\frac{a\hat{i} + b\hat{j} + c\hat{k}}{\sqrt{a^2 + b^2 + c^2}}$

3. 
$$\frac{b\hat{i} + c\hat{j} + a\hat{k}}{\sqrt{a^2 + b^2 + c^2}}$$

4. 
$$\frac{\hat{i} + \hat{j} + \hat{k}}{\sqrt{3}}$$

25. A solid cylinder of height  $H$ , radius  $R$  and density  $\rho$ , floats vertically on the surface of a liquid of density  $\rho_0$ . The cylinder will be set into oscillatory motion when a small instantaneous downward force is applied. The frequency of oscillation is

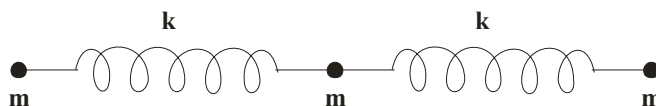
1. 
$$\frac{\rho g}{\rho_0 H}$$

2. 
$$\frac{\rho}{\rho_0} \sqrt{\frac{g}{H}}$$

3. 
$$\sqrt{\frac{\rho g}{\rho_0 H}}$$

4. 
$$\sqrt{\frac{\rho_0 g}{\rho H}}$$

26. Three particles of equal mass  $m$  are connected by two identical massless springs of stiffness constant  $k$  as shown in the figure:



If  $x_1, x_2$  and  $x_3$  denote the horizontal displacements of the masses from their respective equilibrium positions, the potential energy of the system is

1. 
$$\frac{1}{2} k [x_1^2 + x_2^2 + x_3^2]$$

2. 
$$\frac{1}{2} k [x_1^2 + x_2^2 + x_3^2 - x_2(x_1 + x_3)]$$

3. 
$$\frac{1}{2} k [x_1^2 + 2x_2^2 + x_3^2 + 2x_2(x_1 + x_3)]$$

4. 
$$\frac{1}{2} k [x_1^2 + 2x_2^2 + x_3^2 - 2x_2(x_1 + x_3)]$$

27. Let  $V$ ,  $p$  and  $E$  denote the speed, the magnitude of the momentum, and the energy of a free particle of rest mass  $m$ . Then

1. 
$$\frac{dE}{dp} = \text{constant}$$

2. 
$$p = mv$$

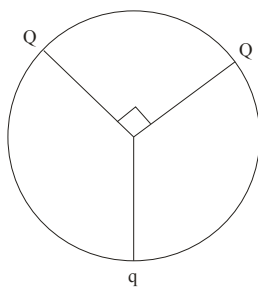
3. 
$$v = \frac{cp}{\sqrt{p^2 + m^2 c^2}}$$

4. 
$$E = mc^2$$

28. A binary star system consists of two stars  $S_1$  and  $S_2$ , with masses  $m$  and  $2m$  respectively separated by a distance  $r$ . If both  $S_1$  and  $S_2$  individually follow circular orbits around the centre of mass with instantaneous speeds  $v_1$  and  $v_2$  respectively, the speeds ratio  $v_1/v_2$  is

- |               |      |
|---------------|------|
| 1. $\sqrt{2}$ | 2. 1 |
| 3. 1/2        | 4. 2 |

29. Three charges are located on the circumference of a circle of radius  $R$  as shown in the figure below. The two charges  $Q$  subtend an angle  $90^\circ$  at the centre of the circle. The charge  $q$  is symmetrically placed with respect to the charges  $Q$ . If the electric field at the centre of the circle is zero, what is the magnitude of  $Q$ ?



- |                 |                |
|-----------------|----------------|
| 1. $q/\sqrt{2}$ | 2. $\sqrt{2}q$ |
| 3. $2q$         | 4. $4q$        |

30. Consider a hollow charged shell of inner radius  $a$  and outer radius  $b$ . The volume charge density is  $\rho(r) = \frac{k}{r^2}$  ( $k$  is a constant) in the region  $a < r < b$ . The magnitude of the electric field produced at distance  $r > a$  is

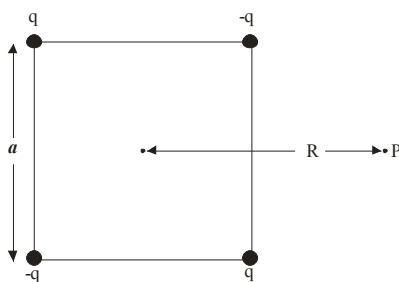
1.  $\frac{k(b-a)}{\epsilon_0 r^2}$  for all  $r > a$
2.  $\frac{k(b-a)}{\epsilon_0 r^2}$  for  $a < r < b$  and  $\frac{kb}{\epsilon_0 r^2}$  for  $r > b$
3.  $\frac{k(b-a)}{\epsilon_0 r^2}$  for  $a < r < b$  and  $\frac{k(b-a)}{\epsilon_0 r^2}$  for  $r > b$
4.  $\frac{k(b-a)}{\epsilon_0 a^2}$  for  $a < r < b$  and  $\frac{k(b-a)}{\epsilon_0 a^2}$  for  $r > b$

31. Consider the interference of two coherent electromagnetic waves whose electric field vectors are given by  $\vec{E}_1 = \hat{i}E_0 \cos \omega t$  and  $\vec{E}_2 = \hat{j}E_0 \cos(\omega t + \varphi)$  where  $\varphi$  is the phase difference. The intensity of the resulting wave is given by  $\frac{\epsilon_0}{2} \langle E^2 \rangle$ , where  $\langle E^2 \rangle$  is the time average of  $E^2$ .

The total intensity is

- |                                      |                                      |
|--------------------------------------|--------------------------------------|
| 1. 0                                 | 2. $\epsilon_0 E_0^2$                |
| 3. $\epsilon_0 E_0^2 \sin^2 \varphi$ | 4. $\epsilon_0 E_0^2 \cos^2 \varphi$ |

32. Four charges (two  $+q$  and two  $-q$ ) are kept fixed at the four vertices of a square of side  $a$  as shown



At the point P which is at a distance  $R$  from the centre ( $R \gg a$ ), the potential is proportional to

- |            |            |
|------------|------------|
| 1. $1/R$   | 2. $1/R^2$ |
| 3. $1/R^3$ | 4. $1/R^4$ |

33. A point charge  $q$  of mass  $m$  is kept at a distance  $d$  below a grounded infinite conducting sheet which lies in the  $xy$ -plane. For what value of  $d$  will the charge remains stationary?

- |                                    |                                 |
|------------------------------------|---------------------------------|
| 1. $q / 4\sqrt{mg\pi\epsilon_0}$   | 2. $q / \sqrt{mg\pi\epsilon_0}$ |
| 3. There is no finite value of $d$ | 4. $\sqrt{mg\pi\epsilon_0} / q$ |

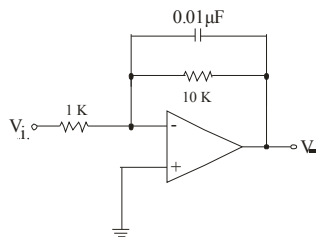
34. The wavefunction of a state of the hydrogen atom is given by

$$\Psi = \psi_{200} + 2\psi_{211} + 3\psi_{210} + \sqrt{2}\psi_{21-1}$$

where  $\psi_{nlm}$  is the normalized eigen function of the state with quantum numbers  $n$ ,  $l$  and  $m$  in the usual notation. The expectation value of  $L_z$  in the state  $\Psi$  is



2.  $\Delta W$  is a perfect differential if the process is adiabatic  
 3.  $\Delta W$  is always a perfect differential  
 4.  $\Delta W$  cannot be a perfect differential
40. Consider a system of three spins  $S_1, S_2$  and  $S_3$  each of which can take values  $+1$  and  $-1$ . The energy of the system is given by  $E = -J[S_1S_2 + S_2S_3 + S_3S_1]$ , where  $J$  is a positive constant. The minimum energy and the corresponding number of spin configurations are, respectively,
1.  $J$  and 1  
 2.  $-3J$  and 1  
 3.  $-3J$  and 2  
 4.  $-6J$  and 2
41. The minimum energy of a collection of 6 non-interacting electrons of spin  $-1/2$  and mass  $m$  placed in a one dimensional infinite square well potential of width  $L$  is
1.  $14\pi^2\hbar^2 / mL^2$   
 2.  $91\pi^2\hbar^2 / mL^2$   
 3.  $7\pi^2\hbar^2 / mL^2$   
 4.  $3\pi^2\hbar^2 / mL^2$
42. A live music broadcast consists of a radio-wave of frequency 7 MHz, amplitude-modulated by a microphone output consisting of signals with a maximum frequency of 10 kHz. The spectrum of modulated output will be zero outside the frequency band
1. 7.00 MHz to 7.01 MHz  
 2. 6.99 MHz to 7.01 MHz  
 3. 6.99 MHz to 7.00 MHz  
 4. 6.995 MHz to 7.005 MHz
43. In the op-amp circuit shown in the figure,  $V_i$  is a sinusoidal input signal of frequency 10 Hz and  $V_o$  is the output signal.



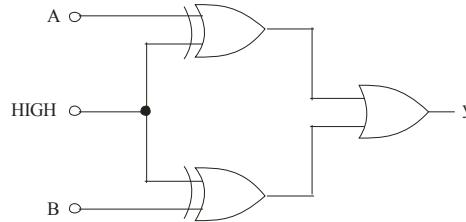
The magnitude of the gain and the phase shift, respectively, close to the values

1.  $5\sqrt{2}$  and  $\pi/2$   
 2.  $5\sqrt{2}$  and  $-\pi/2$

3. 10 and zero

4. 10 and  $\pi$

44. The logic circuit shown in the figure below



Implements the Boolean expression

1.  $y = \overline{A.B}$

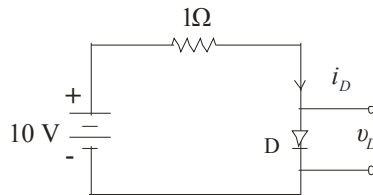
2.  $y = \overline{A.B}$

3.  $y = A.B$

4.  $y = A + B$

45. A diode D as shown in the circuit has an  $i-v$  relation that can be approximated by

$$i_D = \begin{cases} v_D^2 + 2v_D, & \text{for } v_D > 0 \\ 0, & \text{for } v_D \leq 0 \end{cases}$$



The value of  $v_D$  in the circuit is

1.  $(-1 + \sqrt{11})V$

2.  $8V$

3.  $5V$

4.  $2V$

## PART – C

46. The Taylor expansion of the function  $\ln(\cosh x)$ , where  $x$  is real, about the point  $x=0$  starts with the following terms:

- |                                                |                                               |
|------------------------------------------------|-----------------------------------------------|
| 1. $-\frac{1}{2}x^2 + \frac{1}{12}x^4 + \dots$ | 2. $\frac{1}{2}x^2 - \frac{1}{12}x^4 + \dots$ |
| 3. $-\frac{1}{2}x^2 + \frac{1}{6}x^4 + \dots$  | 4. $\frac{1}{2}x^2 + \frac{1}{6}x^4 + \dots$  |

47. Given a  $2 \times 2$  unitary matrix  $U$  satisfying  $U^\dagger U = U U^\dagger = 1$  with  $\det U = e^{i\phi}$ , one can construct a unitary matrix  $V$  ( $V^\dagger V = V V^\dagger = 1$ ) with  $\det V = 1$  from it by

1. multiplying  $U$  by  $e^{-i\phi/2}$
2. multiplying any single element of  $U$  by  $e^{-i\phi}$
3. multiplying any row or column of  $U$  by  $e^{-i\phi/2}$
4. multiplying  $U$  by  $e^{-i\phi}$

48. The value of the integral  $\int_C \frac{z^3 dz}{z^2 - 5z + 6}$ , where  $C$  is a closed contour defined by the equation  $2|z| - 5 = 0$ , traversed in the anti-clockwise direction, is

- |               |              |
|---------------|--------------|
| 1. $-16\pi i$ | 2. $16\pi i$ |
| 3. $8\pi i$   | 4. $2\pi i$  |

49. The function  $f(x)$  obeys the differential equation  $\frac{d^2 f}{dx^2} - (3 - 2i)f = 0$  and satisfies the conditions  $f(0) = 1$  and  $f(x) \rightarrow 0$  as  $x \rightarrow \infty$ . The value of  $f(\pi)$  is

- |                 |                |
|-----------------|----------------|
| 1. $e^{2\pi}$   | 2. $e^{-2\pi}$ |
| 3. $-e^{-2\pi}$ | 4. $-e^{2\pi}$ |

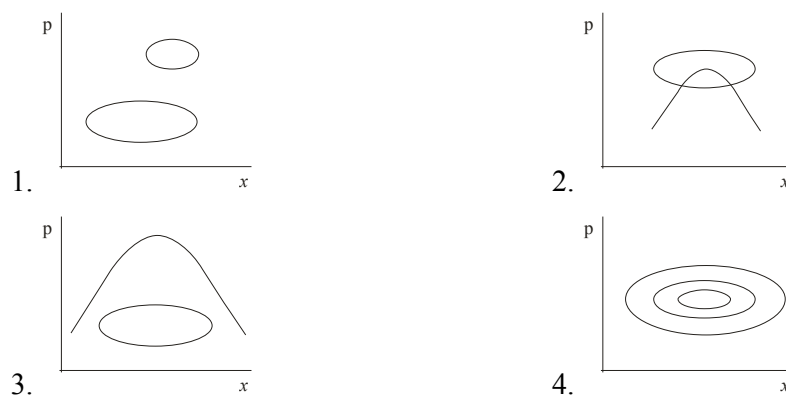
50. A planet of mass  $m$  moves in the gravitational field of the Sun (mass  $M$ ). If the semi-major and semi-minor axes of the orbit are  $a$  and  $b$  respectively, the angular momentum of the planet is:

- |                                   |                                   |
|-----------------------------------|-----------------------------------|
| 1. $\sqrt{2GMm^2(a+b)}$           | 2. $\sqrt{2GMm^2(a-b)}$           |
| 3. $\sqrt{\frac{2GMm^2 ab}{a-b}}$ | 4. $\sqrt{\frac{2GMm^2 ab}{a+b}}$ |

51. The Hamiltonian of a simple pendulum consisting of a mass  $m$  attached to a massless string of length  $l$  is  $H = \frac{p_\theta^2}{2ml^2} + mgl(1 - \cos\theta)$ . If  $L$  denotes the Lagrangian, the value of  $\frac{dL}{dt}$  is

- |                                        |                                         |
|----------------------------------------|-----------------------------------------|
| 1. $-\frac{2g}{l} p_\theta \sin\theta$ | 2. $-\frac{g}{l} p_\theta \sin 2\theta$ |
| 3. $\frac{g}{l} p_\theta \cos\theta$   | 4. $lp_\theta^2 \cos\theta$             |

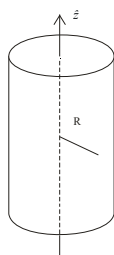
52. Which of the following set of phase-space trajectories is not possible for a particle obeying Hamilton's equations of motion?



53. Two bodies of equal mass  $m$  are connected by a massless rigid rod of length  $l$  lying in the  $xy$ -plane with the centre of the rod at the origin. If this system is rotating about the  $z$ -axis with a frequency  $\omega$ , its angular momentum is

- |                   |                   |
|-------------------|-------------------|
| 1. $ml^2\omega/4$ | 2. $ml^2\omega/2$ |
| 3. $ml^2\omega$   | 4. $2ml^2\omega$  |

54. An infinite solenoid with its axis of symmetry along the  $z$ -direction carries a steady current  
I. The vector potential  $\vec{A}$  at a distance  $R$  from the axis



1. is constant and varies as  $R$  outside the solenoid
2. varies as  $R$  inside and is constant outside the solenoid
3. varies as  $1/R$  inside and as  $R$  outside the solenoid
4. varies as  $R$  inside and as  $1/R$  outside the solenoid

55. Consider an infinite conducting sheet in the  $xy$ -plane with a time dependent current density  $Kt \hat{i}$ , where  $K$  is a constant. The vector potential at  $(x, y, z)$  is given by  $\vec{A} = \frac{\mu_0 K}{4c} (ct - z)^2 \hat{i}$ .

The magnetic field  $\vec{B}$  is

- |                                           |                                           |
|-------------------------------------------|-------------------------------------------|
| 1. $\frac{\mu_0 K t}{2} \hat{j}$          | 2. $-\frac{\mu_0 K z}{2c} \hat{j}$        |
| 3. $-\frac{\mu_0 K}{2c} (ct - z) \hat{i}$ | 4. $-\frac{\mu_0 K}{2c} (ct - z) \hat{j}$ |

56. When a charge particle emits electromagnetic radiation, the electric field  $\vec{E}$  and the Poynting vector  $\vec{S} = \frac{1}{\mu_0} \vec{E} \times \vec{B}$  at a large distance  $r$  from the emitter vary as  $\frac{1}{r^n}$  and  $\frac{1}{r^m}$  respectively.

Which of the following choices for  $n$  and  $m$  are correct?

- |                        |                        |
|------------------------|------------------------|
| 1. $n = 1$ and $m = 1$ | 2. $n = 2$ and $m = 2$ |
| 3. $n = 1$ and $m = 2$ | 4. $n = 2$ and $m = 4$ |

57. The energies in the ground state and first excited state of a particle of mass  $m = \frac{1}{2}$  in a potential  $V(x)$  are  $-4$  and  $-1$ , respectively, (in units in which  $\hbar = 1$ ). If the corresponding wavefunctions are related by  $\psi_1(x) = \psi_0(x) \sinh x$ , then the ground state eigenfunction is

- |                                        |                                  |
|----------------------------------------|----------------------------------|
| 1. $\psi_0(x) = \sqrt{\text{sech } x}$ | 2. $\psi_0(x) = \text{sech } x$  |
| 3. $\psi_0(x) = \text{sech}^2 x$       | 4. $\psi_0(x) = \text{sech}^3 x$ |

58. The perturbation

$$H' = \begin{cases} b(a-x), & -a < x < a \\ 0, & \text{otherwise} \end{cases}$$

acts on a particle of mass  $m$  confined in an infinite square well potential

$$V(x) = \begin{cases} 0, & -a < x < a \\ \infty, & \text{otherwise} \end{cases}$$

The first order correction to the ground-state energy of the particle is

1.  $\frac{ba}{2}$
2.  $\frac{ba}{\sqrt{2}}$
3.  $2ba$
4.  $ba$

59. Let  $|0\rangle$  and  $|1\rangle$  denote the normalized eigenstates corresponding to the ground and the first excited states of a one-dimensional harmonic oscillator. The uncertainty  $\Delta x$  in the state

$$\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$$
 is

1.  $\Delta x = \sqrt{\hbar / 2m\omega}$
2.  $\Delta x = \sqrt{\hbar / m\omega}$
3.  $\Delta x = \sqrt{2\hbar / m\omega}$
4.  $\Delta x = \sqrt{\hbar / 4m\omega}$

60. What would be the ground state energy of the Hamiltonian

$$H = -\frac{\hbar^2}{2m} \frac{d^2}{dx^2} - \alpha \delta(x)$$

if variational principle is used to estimate it with the trial wavefunction  $\psi(x) = Ae^{-bx^2}$  with  $b$

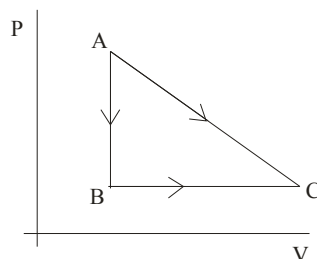
as the variational parameter? [Hint:  $\int_{-\infty}^{\infty} x^{2n} e^{-2bx^2} dx = (2b)^{-n-\frac{1}{2}} \Gamma\left(n + \frac{1}{2}\right)$ ]

1.  $-m\alpha^2 / 2\hbar^2$
2.  $-2m\alpha^2 / \pi\hbar^2$
3.  $-m\alpha^2 / \pi\hbar^2$
4.  $m\alpha^2 / \pi\hbar^2$

61. The free energy difference between the super conducting and the normal states of a material is given by  $\Delta F = F_S - F_N = \alpha|\psi|^2 + \frac{\beta}{2}|\psi|^4$ , where  $\psi$  is an order parameter and  $\alpha$  and  $\beta$  are constants such that  $\alpha > 0$  in the normal and  $\alpha < 0$  in the superconducting state, while  $\beta > 0$  always. The minimum value of  $\Delta F$  is

1.  $-\alpha^2 / \beta$
2.  $-\alpha^2 / 2\beta$
3.  $-3\alpha^2 / 2\beta$
4.  $-5\alpha^2 / 2\beta$

62. A given quantity of gas is taken from the state  $A \rightarrow C$  reversibly, by two paths,  $A \rightarrow C$  directly and  $A \rightarrow B \rightarrow C$  as shown in the figure below.



During the process  $A \rightarrow C$  the work done by the gas is 100J and the heat absorbed is 150J.

If during the process  $A \rightarrow B \rightarrow C$  the work done by the gas is 30 J, the heat absorbed is

- |          |          |
|----------|----------|
| 1. 20 J  | 2. 80 J  |
| 3. 220 J | 4. 280 J |
63. Consider a one-dimensional Ising model with  $N$  spins, at very low temperatures when almost all the spins are aligned parallel to each other. There will be a few spin flips with each flip costing an energy  $2J$ . In a configuration with  $r$  spin flips, the energy of the system is  $E = -NJ + 2rJ$  varies from 0 to  $N$ . The partition function is

- |                                           |                                           |
|-------------------------------------------|-------------------------------------------|
| 1. $\left(\frac{J}{k_B T}\right)^N$       | 2. $e^{-NJ/k_B T}$                        |
| 3. $\left(\sinh \frac{J}{k_B T}\right)^N$ | 4. $\left(\cosh \frac{J}{k_B T}\right)^N$ |

64. A magnetic field sensor based on the Hall effect is to be fabricated by implanting As into a Si film of thickness  $1 \mu\text{m}$ . The specifications require a magnetic field sensitivity of 500 mV/Tesla at an excitation current of 1 mA. The implantation dose is to be adjusted such that the average carrier density, after activation is

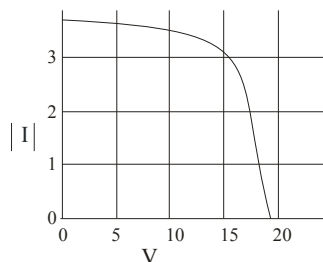
- |                                         |                                         |
|-----------------------------------------|-----------------------------------------|
| 1. $1.25 \times 10^{26} \text{ m}^{-3}$ | 2. $1.25 \times 10^{22} \text{ m}^{-3}$ |
| 3. $4.1 \times 10^{21} \text{ m}^{-3}$  | 4. $4.1 \times 10^{20} \text{ m}^{-3}$  |

65. Band-pass and band-reject filters can be implemented by combining a low pass and a high pass filter in series and in parallel, respectively. If the cut-off frequencies of the low pass and

high pass filters are  $\omega_0^{LP}$  and  $\omega_0^{HP}$ , respectively, the condition required to implement the band-pass and band-reject filters are, respectively,

1.  $\omega_0^{HP} < \omega_0^{LP}$  and  $\omega_0^{HP} < \omega_0^{LP}$
2.  $\omega_0^{HP} < \omega_0^{LP}$  and  $\omega_0^{HP} > \omega_0^{LP}$
3.  $\omega_0^{HP} > \omega_0^{LP}$  and  $\omega_0^{HP} < \omega_0^{LP}$
4.  $\omega_0^{HP} > \omega_0^{LP}$  and  $\omega_0^{HP} > \omega_0^{LP}$

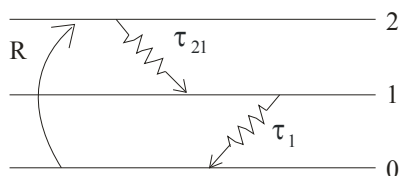
66. The output characteristic of a solar panel at a certain level of irradiance is shown in the figure below.



If the solar cell is to power a load of  $5\Omega$ , the power drawn by the load is

1. 97 W
2. 73 W
3. 50 W
4. 45 W

67. Consider the energy level diagram shown below, which corresponds to the molecular nitrogen laser.



If the pump rate  $R$  is  $10^{20}$  atoms  $cm^{-3}s^{-1}$  and the decay routes are as shown with  $\tau_{21} = 20$  ns and  $\tau_1 = 1\mu s$ , the equilibrium populations of states 2 and 1 are, respectively,

1.  $10^{14} cm^{-3}$  and  $2 \times 10^{12} cm^{-3}$
2.  $2 \times 10^{12} cm^{-3}$  and  $10^{14} cm^{-3}$
3.  $2 \times 10^{12} cm^{-3}$  and  $2 \times 10^6 cm^{-3}$
4. Zero and  $10^{20} cm^{-3}$

68. Consider a hydrogen atom undergoing a  $2P \rightarrow 1S$  transition. The lifetime  $t_{sp}$  of the 2P state for spontaneous emission is 1.6 ns and the energy difference between the levels is 10.2 eV .

Assuming that the refractive index of the medium  $n_0 = 1$ , the ratio of Einstein coefficients for stimulated and spontaneous emission  $B_{21}(\omega)/A_{21}(\omega)$  is given by

- |                                             |                                       |
|---------------------------------------------|---------------------------------------|
| 1. $0.683 \times 10^{12} m^3 J^{-1} s^{-1}$ | 2. $0.146 \times 10^{-12} J s m^{-3}$ |
| 3. $6.83 \times 10^{12} m^3 J^{-1} s^{-1}$  | 4. $1.463 \times 10^{-12} J s m^{-3}$ |

69. Consider a He-Ne laser cavity consisting of two mirrors of reflectivities  $R_1 = 1$  and  $R_2 = 0.98$ .

The mirrors are separated by a distance  $d = 20$  cm and the medium in between has a refractive index  $n_0 = 1$  and absorption coefficient  $\alpha = 0$ . The values of the separation between the modes  $\delta\nu$  and the width  $\Delta\nu_p$  of each mode of the laser cavity are:

- |                                                                    |                                                                    |
|--------------------------------------------------------------------|--------------------------------------------------------------------|
| 1. $\delta\nu = 75 \text{ kHz}$ , $\Delta\nu_p = 24 \text{ kHz}$   | 2. $\delta\nu = 100 \text{ kHz}$ , $\Delta\nu_p = 100 \text{ kHz}$ |
| 3. $\delta\nu = 750 \text{ MHz}$ , $\Delta\nu_p = 2.4 \text{ MHz}$ | 4. $\delta\nu = 2.4 \text{ MHz}$ , $\Delta\nu_p = 750 \text{ MHz}$ |

70. Non-interacting bosons undergo Bose-Einstein Condensation (BEC) when trapped in a three-dimensional isotropic simple harmonic potential. For BEC to occur, the chemical potential must be equal to

- |                     |                  |
|---------------------|------------------|
| 1. $\hbar\omega/2$  | 2. $\hbar\omega$ |
| 3. $3\hbar\omega/2$ | 4. 0             |

71. In a band structure calculation, the dispersion relation for electrons is found to be

$$\varepsilon_k = \beta(\cos k_x a + \cos k_y a + \cos k_z a)$$

where  $\beta$  is a constant and  $a$  is the lattice constant. The effective mass at the boundary of the first Brillouin zone is

- |                                  |                                  |
|----------------------------------|----------------------------------|
| 1. $\frac{2\hbar^2}{5\beta a^2}$ | 2. $\frac{4\hbar^2}{5\beta a^2}$ |
| 3. $\frac{\hbar^2}{2\beta a^2}$  | 4. $\frac{\hbar^2}{3\beta a^2}$  |

72. The radius of the Fermi sphere of free electrons in a monovalent metal with an fcc structure, in which the volume of the unit cell is  $a^3$ , is

1.  $\left(\frac{12\pi^2}{a^3}\right)^{1/3}$

2.  $\left(\frac{3\pi^2}{a^3}\right)^{1/3}$

3.  $\left(\frac{\pi^2}{a^3}\right)^{1/3}$

4.  $\frac{1}{a}$

73. The muon has mass  $105 \text{ MeV}/c^2$  and mean lifetime  $2.2\mu\text{s}$  in its rest frame. The mean distance traversed by a muon of energy  $315 \text{ MeV}/c^2$  before decaying is approximately

1.  $3 \times 10^5 \text{ km}$

2.  $2.2 \text{ cm}$

3.  $6.6 \mu\text{m}$

4.  $1.98 \text{ km}$

74. Consider the following particles: the proton  $p$ , the neutron  $n$ , the neutral pion  $\pi^0$  and the delta resonance  $\Delta^+$ . When ordered in terms of decreasing lifetime, the correct arrangement is as follows:

1.  $\pi^0, n, p, \Delta^+$

2.  $p, n, \Delta^+, \pi^0$

3.  $p, n, \pi^0, \Delta^+$

4.  $\Delta^+, n, \pi^0, p$

75. The single particle energy difference between the p-orbitals (i.e.,  $p_{3/2}$  and  $p_{1/2}$ ) of the nucleus  ${}_{50}^{114}\text{Sn}$  is  $3 \text{ MeV}$ . The energy difference between the states in its  $1f$  orbital is

1.  $-7 \text{ MeV}$

2.  $7 \text{ MeV}$

3.  $5 \text{ MeV}$

4.  $-5 \text{ MeV}$

